TEACHERS' RESPONSES TO A COMMON SET OF HIGH POTENTIAL INSTANCES OF STUDENT MATHEMATICAL THINKING

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This study investigates teacher responses to a common set of high potential instances of student mathematical thinking to better understand the role of the teacher in shaping meaningful mathematical discourse in their classrooms. Teacher responses were coded using a scheme that disentangles the teacher move from other aspects of the teacher response, including who the response is directed to and the degree to which the student thinking is honored. Teachers tended to direct their response to the student who had shared their thinking and to explicitly incorporate ideas core to the student thinking in their response. We consider the nature of these responses in relation to principles of productive use of student mathematical thinking.

Keywords: Classroom Discourse, Instructional Activities and Practices

Recommendations for effective mathematics teaching stress the importance of engaging students in meaningful mathematical discourse (e.g., National Council of Teachers of Mathematics [NCTM], 2014). Research has begun to help us understand how to effectively orchestrate discourse around written records of student work (e.g., Stein, Engle, Smith, & Hughes, 2008), but much less is known about how to effectively use the in-the-moment mathematical thinking that emerges during classroom mathematics discourse. One issue related to responding to student thinking is that not all student thinking warrants the same consideration. Rather, student thinking varies in the degree to which it provides leverage for accomplishing mathematical goals. Leatham, Peterson, Stockero, and Van Zoest (2015) described a framework to identify those instances of student thinking—MOSTs—that provide such leverage, but little is known, as of yet, about effective responses to MOSTs. The study reported here investigated teacher responses to a common set of MOSTs. Better understanding such responses will contribute to better understanding the role of the teacher in shaping meaningful mathematical discourse in their classrooms.

Literature Review

Research on classroom discourse has identified patterns in teachers' responses to student thinking. Mehan (1979) coined *IRE*—Initiation, Response, Evaluation—to describe a common pattern of classroom interaction where the teacher's main follow-up to an elicited student response is to evaluate it. An IRE interaction is an example of what Wood (1998) referred to as *funneling,* where the teacher's response is intended to corral students' thinking within predetermined and often narrowly-defined parameters. By contrast, Wood characterized certain other teacher responses as *focusing*; in these responses a teacher "keep[s] attention focused on the discriminating aspects of the solution" (p. 175).

Van Zee and Minstrell (1997) explored what they called a *reflective toss*—a pattern that consists of a student statement, teacher question, and additional student statements. Van Zee and Minstrell argued that changing the evaluation component of IRE to a question could positively impact the nature of classroom discourse by changing students' expectations for participation. These results are

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not unique; in general, research has found that teacher responses matter. Fennema et al. (1996) found that increases in teachers' focus on student thinking in their classrooms were directly related to improvements in their students' achievement. Kazemi and Stipek's (2001) investigation revealed that teachers in high-press classrooms—classrooms in which the teacher responded to their students' contributions to classroom discourse by pressing the students to further engage in thinking about important mathematics in their contributions—provided their students with increased learning opportunities.

Other researchers have looked at collections of teacher moves that accomplish a particular purpose related to student thinking. Lineback (2015), for example, investigated the construct of *redirection*—"instances when a teacher invites students to shift or *redirect* their attention to a new locus" (p. 419). This work generated a taxonomy of redirections to deconstruct teacher responses and analyze the contribution of different redirection responses to instruction. Bishop, Hardison, and Przybyla-Kuchek (2016) described the mathematical contributions of students, the moves teacher made in response, and the relationship between these contributions and moves, through the lens of *responsiveness*, which they defined as the extent to which teacher responses "mutually acknowledge, take up, and reflect an awareness of student thinking" (p. 1173). Connor, Singletary, Smith, Wagner, and Francisco (2014) developed a framework that includes teacher responses to student thinking that support collective argumentation in the classroom. Their work provides important information for focusing on a particular type of student thinking—that which involves mathematical argumentation.

In the work reported here, we narrow down the type of student thinking to MOSTs and consider the extent to which the teacher responses to those MOSTs accomplish the purpose of *building* on them.

Theoretical Framework

MOSTs (Leatham et al., 2015) are instances of student thinking worth *building* on—that is, "student thinking worth making the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea" (Van Zoest et al., 2017, p. 36). To take full advantage of these opportune instances of student thinking, one would want to seek to build on MOSTs *in the moment*. Such use encapsulates the core ideas of current thinking about effective teaching and learning of mathematics (e.g., NCTM, 2014), including that student mathematics is at the forefront and that students are positioned as legitimate mathematical thinkers, engaged in sense making, and working collaboratively. These ideas serve as the principles underlying our conceptualization of productive use of MOSTs (see Figure 1).

1. The mathematics of the MOST is at the forefront.
2. Students are positioned as legitimate mathematical thinkers.
3. Students are engaged in sense making.
4. Students are working collaboratively.

Figure 1. Principles underlying productive use of MOSTs (Van Zoest et al., 2016).

We theorize that building on MOSTs is a particularly productive way for teachers to engage students in meaningful mathematical learning. Van Zoest, Peterson, Leatham, & Stockero (2016) put forth a conceptualization of the teaching practice of building on MOSTs (see Figure 2). Together the principles (see Figure 1) and building subpractices (see Figure 2) provide a way to assess the extent to which teacher responses to MOSTs instantiate the practice of building.

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1. Make the object of consideration clear (make precise)
2. Turn the object of consideration over to the students with parameters that put them in a sense-making situation (grapple toss)
3. Orchestrate a whole-class discussion in which students collaboratively make sense of the object of consideration (orchestrate)
4. Facilitate the extraction and articulation of the mathematical point of the object of consideration (make explicit)

Figure 2. Sequence of subpractices of the teaching practice of building on MOSTs.

Methodology

The Scenario Interview (Stockero et al., 2015) is a tool to investigate how teachers think about responding to student thinking during instruction. During the interview teachers are presented with instances of mathematical thinking from eight individual students—four each from an algebra and a geometry context. The interviewee is situated as the teacher and asked to describe what they might do next were the instance to occur in their mathematics classroom and to explain why they would respond in that way. The Scenario Interview allowed us to compare teacher responses to a common set of student thinking. The analysis reported here focuses on responses to the four instances, two from each context, in which the student thinking was a MOST. The four MOSTs and their contexts are provided in Figure 3.

Scenario	Context	MOST
G1	Students were sharing their solutions to the following task (a corresponding picture was on the board). *Given two concentric circles, radii 5cm and 3cm, what is the area of the band between the circles?*	Chris shared his solution: "The radius of the big circle is 5 and the radius of the little circle is 3, so the gap is 2, so the area of the band is 4π cm^2."
G3		Pat explained how he got the same answer as Chris (4π cm^2) a different way: "π times r^2 for the big circle is π times 5^2, which is 10π and π times 3^2 is 6π for the little circle. I minused (sic) them and got 4π as my answer."
A2	Students had been discussing the following task and had come up with the equation $y = 10x + 25$. *Jenny received $25 for her birthday that she deposited into a savings account. She has a babysitting job that pays $10 per week, which she deposits into her account each week. Write an equation that she can use to predict how much she will have saved after any number of weeks.*	Casey said, "You could also change the story so the number in front of the x is negative."
A3	The teacher asked, "How do we find the equation given any table?" and put this generic table of values [to the right] on the board for the students to use in their explanation. X | Y 0 | 15 2 | 19 3 | 21 5 | 25	Jamie said, "I found the number in front of the x by subtracting the y-values in the table, 21 - 19, so that number is 2."

Figure 3. MOSTs that formed the basis of the teacher responses and their contexts.

Data Analysis

The data for this study consisted of video recorded interviews with 25 secondary school mathematics teachers from several sites across the United States. These teachers were representative of a set of 44 teachers who participated in our larger project. We used Studiocode (SportsTec, 1997-

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2015) video analysis software to segment each interview into the instances of student thinking and the teacher responses to each individual instance—everything a teacher said about how they would respond to that instance. Transcriptions of the videos were used to facilitate the analysis. For the 4 instances and 25 teachers of this study, there were a total of 100 teacher responses. In one of those responses the teacher did not provide a description of how they would respond to the instance because they were not able to envision it happening in their classroom, thus 99 teacher responses were analyzed for this study.

The resulting teacher responses were then coded using the *Teacher Response Coding Scheme (TRC)* (Peterson et al., in press), a scheme that disentangles the teacher move from other aspects of the teacher response, including the Actor and the degree to which the student thinking is honored (Recognition-Action and Recognition-Idea). Figure 4 provides the TRC coding categories and codes that were included in this analysis.

Category	Coding Category Description	Codes
Actor	Who is publicly asked to consider the student thinking	teacher, same student(s), other student(s), whole class
Recognition-Action	The degree to which the teacher response uses the student action, either verbal (words) or non-verbal (gestures or work)	explicit, implicit, or not
Recognition-Idea	The extent to which the student is likely to recognize their idea in the teacher response	core, peripheral, other, cannot infer, not applicable
Move	What the actor is doing or being asked to do with respect to the instance of student thinking	adjourn, allow, check-in, clarify, collect, connect, correct, develop, dismiss, evaluate, justify, literal, repeat, validate

Figure 4. Subset of the *Teacher Response Coding Scheme* (*TRC*) used in this paper.

Results and Discussion

We discuss findings related to specific aspects of teachers' responses to MOSTs as well as interactions among those aspects. We first focus on the Actor and Move and their interactions, followed by the individual Recognition categories and their interactions. In doing so, we highlight how a response might adhere to the principles underlying productive use of MOSTs or contribute to enacting subpractices of building.

Actor and Move

With respect to the actor, the majority of teacher responses (66%) had the *same student* as the actor, meaning that the teacher proposed a move that was directed back to the student who had contributed the original thinking (see Table 1). In about 24% of the instances, the teacher move was directed to the *whole class*.

With respect to the moves, two occurred much more frequently than the others; together, *develop* (37%) and *justify* (18%) moves accounted for over half of the data. In a develop move, the teacher provides or asks for an expansion of the student thinking that goes beyond a simple clarification. In a justify move, the teacher asks for or provides a justification of the instance. Since our data showed that these moves had a *same student* or *whole class* actor, in both cases the teacher was asking for, rather than providing, the expansion or justification.

Table 1: Actor and Move

	Same Student	Whole Class	Teacher	Other Student(s)	TOTAL
Adjourn	0	0	3	0	3
Allow	0	3	2	1	6
Clarify	5	0	0	0	5
Collect	2	4	0	1	7
Connect	1	4	0	1	6
Correct	1	0	0	0	1
Develop	32	5	0	0	37
Dismiss	0	0	1	0	1
Evaluate	0	4	0	0	4
Justify	16	2	0	0	18
Literal	4	2	0	0	6
Repeat	4	0	0	1	5
TOTAL	65	24	6	4	99

Taken together, the Actor and Move findings suggest that teachers might instinctively respond to MOSTs by asking the student who provided the thinking to either expand upon or justify their idea. Because MOSTs are instances that a teacher can build upon to "engage the class in making sense of [student] thinking to better understand an important mathematical idea" (Van Zoest et al., 2017, p. 36), however, asking the student to develop or justify their idea may not always be necessary and may actually limit students' opportunities to make sense of mathematical ideas. For example, consider scenario A2 (see Figure 3). It turned out that nearly half of the instances of *develop* moves with *same student* actor (15 of 32) occurred in response to this scenario. The most common teacher move in this instance was to ask Casey, the student who made the suggestion, to explain how they would change the story (e.g., "Well what do you mean? What sort of an equation, or what sort of a real life situation can you think of where that would be a negative?" (Teacher 6 [T6]). Contrast this response with a similar one directed instead to the whole class: "Interesting comment… who can come up with a story, a situation that would match what Casey is saying?" (T7). In this case, we would argue that directing the response to the whole class might be more productive, as it would engage all of the students in trying to come up with a situation where the coefficient is negative, likely advancing the entire class's understanding of the mathematics of linear equations.

Similarly, consider scenario A3. More than two-thirds of the *justify* moves with *same student* actor (11 of 16) occurred in response to this scenario. The most common response to this instance was to ask Jamie why they used the numbers that they did (e.g., "Why did you do the 21 minus the 19? Why didn't you do the 19 minus the 15?" (T14)). This response would allow Jamie to justify their idea, but does not engage the whole class in thinking about the importance of taking into account the differences between x-values as well as the y-values when calculating the rate of change. Consider an alternate response directed to the whole class, such as: "So [Jamie] got 2 from subtracting those two numbers, so what if I pick 19 and 15? If I subtract those, I get 4. Why did we get two different answers?" (T21). Such a response would allow all of the students to consider the mathematics of rate of change. We argue that teachers who respond to MOSTs by asking the student who shared the original thinking for justification may be focused on the details of the situation, whereas those who ask the whole class for justification may be more focused on the big mathematical picture.

In general, responses that turn the mathematics of a MOST over to the whole class instead of

engaging a single student better adhere to principles underlying productive use of MOSTs. Such responses provide all students the opportunity to collaboratively engage in making sense of the mathematics of the MOST. In doing so, they put the students' mathematics at the forefront and position all students as legitimate mathematical thinkers. These responses may also demonstrate an ability to discriminate between those instances that need to be made precise before the teacher can turn them over to students (grapple toss) and those that do not.

Although the goal of building on MOSTs is to have the whole class consider the student mathematics of the instance, there are some cases where directing the initial teacher response back to the same student might be desirable. For example, in scenario G1, it is quite possible that other students in the class would not initially understand Chris' explanation, so the most common teacher response in our data, "ask him to explain by using…pictures and words, like how he came up with the 4π" (T18) may be the teacher helping to make Chris' idea precise before other students are asked to consider it. A move such as this could be an instantiation of the first subpractice of building (make precise)—an important first step in setting the teacher up to engage in the next building subpractice (grapple toss), in which they turn the now-precise student thinking over to the class for consideration.

Recognition of Student Actions and Ideas

The Recognition codes operationalize the extent to which the student who provided the instance would recognize their thinking in the teacher's response. As seen in Table 2, the majority of teacher responses either *explicitly* (54%) or *implicitly* (32%) incorporated the student's words (verbal) or gestures or work (non-verbal). Only 13% of responses would likely *not* be recognizable to the student as incorporating their own actions. Moreover, the vast majority of the responses (75%) remained *core* to the idea in the instance of student thinking. Together the results indicate that a large percentage of the teacher responses were both *explicit* and *core* (43%), meaning that the teachers in this study honored the student thinking by explicitly incorporating the student's verbal or non-verbal actions and staying focused on the student's core ideas in their described response. For example, the response to scenario G1, "I would want to know what he means by gap. Um, and maybe have him illustrate that visually, just to kind of picture that as a class," (T4) is *explicit* and *core* as it incorporates both the student's words (gap) and his ideas (having him illustrate his idea visually). A response such as this aligns with the principles underlying productive use of MOSTs, as it positions the student as a legitimate mathematical thinker by keeping the students' mathematics at the forefront. In general, many teacher responses that are *core* to the student ideas and *implicitly* incorporate student actions also adhere to the same principles, but may be problematic in that it may not be clear to the student(s) what mathematics is under consideration. For example, the response to scenario A3, "So I would want to ask her, 'Why did you do this? What are you thinking? Tell us a little bit more,'" (T24) fails to specify what mathematics the teacher wants to know more about. Among other things, the teacher could be wondering why the student subtracted or why they chose to select the numbers that they did.

Table 2: Recognition of Student Actions and Ideas

		Student Ideas			
		Core	Peripheral	CNI, Other, N/A	TOTAL
Student Actions	Explicit	43	10	1	54
	Implicit	26	4	2	32
	Not	5	1	7	13
	TOTAL	74	15	10	99

Conclusion

Our findings revealed that the teachers in this study most often responded to MOSTs by making a

develop or *justify* move that was directed to the *same student* who had shared the initial thinking. Additionally, they did so in ways that stayed *core* to the ideas in the student thinking and often *explicitly* incorporated the students' actions.

Responses that either explicitly or implicitly incorporate core ideas of a student's contribution signal that these teachers value the students' contributions. We also see such responses positioning the students as legitimate mathematical thinkers who can make valid contributions to the development of the mathematics in the classroom. Hence the words and idea(s) teachers use in their responses to students' ideas could matter in terms of how students are positioned in the classroom. When the student action that is being considered is explicit, it is easier for the whole class to recognize that student thinking is being honored.

MOSTs are prime opportunities for teachers to enact the building practice, but teachers' tendencies to direct their responses to the student who had shared their idea could prevent them from doing so. As we have illustrated, directing a response to the same student could be productive in cases where the student's idea needs to be made precise before others can consider the idea, but many MOSTs do not require clarification. In these instances, rather than going back to the student, it would be more productive to *toss* the already precise student thinking to the whole class to provide all students an opportunity to collaboratively make sense of the mathematics.

The findings of this study advance research on teachers' in-the-moment responses to student mathematical thinking by moving beyond looking at what moves teachers make, to considering to whom those moves are directed and to what extent those moves would allow students' ideas to be recognizable to them or other students. In doing so, the study builds on the approaches taken in past research on teacher responses to explore more refined approaches that allow the field to look at teacher responses in new ways. Decomposing teacher responses in the way we have in this study has the potential to help teacher educators and researchers focus their development efforts. For example, if the majority of a teacher's responses honor student thinking, but engage only the student who contributed the instance, professional development work with the teacher could focus specifically on understanding the potential in directing a response to the whole class, and when it would and would not be appropriate to do so. Focused efforts such as this would allow professional developers to leverage teachers' strengths and thus develop teachers' practice more effectively.

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